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Optimum Phosphorus Fertilization

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Fertilizer application rate decisions are ubiquitous for Corn Belt farmers. Typically, farmers have their soils analyzed by a private or public laboratory, receive a nutrient application recommendation, and base their actual nutrient application rates on these recommendations. However, recommendations lack explicit recognition of product prices or fertilizer prices. The purpose of this paper is to present a methodology for incorporating prices in recommendations and to examine the impact of prices on the economic optimum application of phosphorus on corn, wheat, and soybeans.

Phosphorus is examined because it has several interesting characteristics. First, its price has changed dramatically during the past decade. The average price paid by U.S. farmers was \$0.085 per pound of P_2O_5 in 1972. By 1980, this price had jumped to \$0.28 per pound. As product prices and fertilizer demand declined during 1981-82, P_2O_5 prices fell and averaged \$0.235 per pound in late 1982 (U.S. Department of Agriculture). Second, soil phosphorus level and phosphorus fertilizer are imperfect substitutes. High phosphorus application rates can substitute for low soil phosphorus levels, and as phosphorus is applied in excess of crop usage, soil phosphorus levels increase. Similarly, low phosphorus applications may be economically optimal as high soil phosphorus levels are "mined." There is even the possibility of the economic optimum application rate being zero; high soil levels of phosphorus would substitute for phosphorus application and soil levels would be drawn down.

Finally, phosphorus is the nutrient primarily responsible for the degradation of many water bodies, such as Lake Erie. It stimulates excessive plant growth. As these plants die, oxygen is depleted, and fish and other aquatic life are stressed (U.S. Army Corps of Engineers). The judicious use of phosphorus would not only assist farmers, but also it might improve water quality for downstream water users.

Crop Response to Phosphorus

There is an absence of nutrient substitution among macro nutrients. Thus, it is possible to consider one macro nutrient independent of others as long as the other nutrients are at some minimum level (Bray; Lanzer and Paris; Perrin; Stauber, Burt, and Linse; Hildreth). In essence the shape of the phosphorus response function is considered constant over the range of economic optimum application rates for other macro nutrients. For Ohio conditions, phosphorus response functions have been estimated for corn, soybeans, and wheat as follows:

$$\log \frac{A - Y_t}{A} = a_1 X_t + a_2 Z_t \tag{1}$$

where A is a maximum yield plateau. In year t, the yield, Y_t, approaches this maximum yield asymptotically at high phosphorus application rates, Z_t, and/or at high soil phosphorus levels X_t. This functional form was proposed by Bray and is now widely accepted by agronomists. The estimated functions are from Johnson and are as follows:

Corn $\quad \log \frac{A - Y_t}{A} = -0.043\ X_t - 0.0091\ Z_t$ where A = 164

Soybeans $\quad \log \frac{A - Y_t}{A} = -0.054\ X_t - 0.0071\ Z_t$ where A = 46.5

Wheat $\quad \log \frac{A - Y_t}{A} = -0.031\ X_t - 0.01\ Z_t$ where A = 60.9

and A are in terms of Y_t bushels per acre, X_t is pounds of P_2O_5 per acre, and Z_t is the pounds of phosphorus per acre in the soil (Bray P_1). The soil The soil phosphorus level tends to decay over time when no phosphorus is applied. As this decay occurs, soil phosphorus levels approach an equilibrium soil test where the amount of decay is equal to plant uptake (Cox, Kamprath and McCollum). With the addition of phosphorus fertilizer, the soil test increases at some proportion of the phosphorus applied (build up rate). The functional form is

$$X_t = X_{eq} + (X_o + F - X_{eq}) \exp(-kt) \tag{2}$$

where X_t is the soil phosphorus level in year t; X_o is the soil phosphorus level in year 0; $F = \sum_{s=0}^{t} bZ_s$ or the sum of all fertilizer added since year 0 times the build up rate, b; X_{eq} is the equilibrium soil test value; and k is the soil test decay rate. For typical Corn Belt soils, b is estimated to be 0.225, k is estimated at 0.1, and X_{eq} at 10 (Johnson; Logan).

Economic Optimum Application Rates

The model used in the analysis is based on short run profit maximization. Each year the farmer optimizes profits by equating the marginal value product of phosphorus with its marginal cost. The model allows substitution between the resource inventory or stock, soil phosphorus level (X_t), and the resource flow, phosphorus application rate (Z_t).

This myopic profit maximization model could be extended to a multiperiod optimization model. With this extension, the problem's complexity expands. There is the uncertainty surrounding future output prices, input prices and appropriate discount rate which surrounds any multiperiod model. More important, another decision variable, the crop to be grown or the rotation to be used, enters the problem. Each period, one state variable (soil phosphorus level) and two decision variables (phosphorus application rate and crop to be grown) would be involved. Dynamic programming might be used to solve the problem (Bellman).

However, it is possible to achieve a global objective by maximizing a sequence of myopic short run objectives (Day et al.). An analogy provided by Tesfatsion is the chess player, myopically attempting to achieve a global checkmate objective through a series of best moves, with each move taking into account the current board configuration.

When period-by-period returns exhibit positive correlation, myopic net return maximization results in global maximum returns (Tesfatsion). That is, multiperiod decision making model is an unnecessary complication. To the extent that annual net returns from cropland are positively correlated, the procedure incorporated in this paper should maximize long run profit maximization objectives.

Substituting the soil test function (2) in the production function (1)

$$\log \frac{A - Y_t}{A} = a_1[X_{eq} + (X_o + F - X_{eq}) \exp(-kt)] + a_2 Z_t$$

Substituting u for the right side of the equation

$$\log \frac{A - Y_t}{A} = u \quad \text{or} \quad Y_t = A(1-10^u) \tag{3}$$

The profit maximizing application rate of phosphorus is found by differentiating equation (3) with respect to Z_t, setting this equal to the ratio of the fertilizer price to the product price (P_{Zt}/P_{Yt}), and solving for the optimum application rate (Z_t) and yield (Y_t). First, differentiating equation (3) with respect to Z_t

$$\frac{d\ Y_t}{d\ Z_t} = (Y_t - A) \cdot \frac{du}{d\ Z_t} \cdot \ln 10$$

$$= (Y_t - A) \cdot [a_1 \cdot b \cdot \exp(-kt) + a_2] \cdot \ln 10$$

Setting this result equal to the price ratio,

$$(Y_t - A) \cdot [a_1 \cdot b \cdot \exp(-kt) + a_2] \cdot \ln 10 = P_{Zt}/\ P_{Yt}$$

Substituting C_1 and C_2 for the two constant terms

$$(Y_t - A) \cdot C_1 \cdot C_2 = P_{Zt}/P_{Yt}$$

Therefore, at the profit maximizing rate of phosphorus application, the yield is

$$Y_t = \frac{P_{Zt}/P_{Yt}}{C_1 \cdot C_2} + A \tag{4}$$

To find the economically optimum phosphorus application rate, equation (4) is solved for Z_t. To find Z_t,

$$A\ (1-10^u) = \frac{P_{Zt}/P_{Yt}}{C_1 \cdot C_2} + A$$

or

$$10^u = - \frac{P_{Zt}/P_{Yt}}{C_1 \cdot C_2}\ /\ A$$

or after dividing u into 2 parts

$$10^{\{a_1(X_{eq} + [X_o + \sum_{s=1}^{t-1} b\ Z_s - X_{eq}]\ \exp(-kt)\}} \cdot 10^{C_1 Z_t} = - \frac{P_{Zt}/P_{Yt}}{C_1 \cdot C_2}\ /\ A$$

From this equation the solution can be derived (Equation (5)). The constant S is substituted for the expression $10^{\{\ \}}$ above.

$$Z_t = \log \left(- \frac{P_{Zt}/P_{Yt}}{C_1 \cdot C_2} \cdot \frac{1}{A} \cdot \frac{1}{S} \right) /\ C_1 \tag{5}$$

According to equation (5), product prices (P_{Yt}) and phosphate price (P_{Zt}) affect the optimum application of phosphorus. Phosphate's relative price is defined by the ratio, P_{Zt}/P_{Yt}. The relative price elasticity is the percentage change in optimum phosphate usage in response to a percentage change in phosphate's relative price, as indicated in equation (6).

$$E = \frac{d\ Z_t}{d\ (P_{Zt}/P_{Yt})} \cdot \frac{P_{Zt}/P_{Yt}}{Z_t} \tag{6}$$

Substituting the right side of equation (5) for Z_t and solving equation (6) produces -

$$E = \frac{1}{C_1} \cdot \frac{1}{Z_t} \cdot \log_{10} e \tag{7}$$

Analysis for a Range of Soil Phosphorus Levels

The optimum phosphate application rate (defined in equation (5)) and the relative price elasticity of phosphate usage (defined in equation (7)) are calculated for a range of soil phosphorus levels and crops. Crops considered are corn (Figure 1), wheat (Figure 2), and soybeans (Figure 3). Soil phosphorus levels are Bray phosphorus tests of 20, 40, and 60 pounds of phosphorus per acre.

For corn, the economic application rate is calculated assuming a product price of \$2.50 per bushel and a \$0.25 per pound phosphate (P_2O_5) price. As illustrated in Figure (1), Panel (a), the optimum phosphate application rate is zero on soils with high phosphorus levels. The soil is essentially "mined" until the Bray phosphorus test is approximately 40 pounds per acre. Then, over time, phosphate is applied in increasing quantities and the phosphorus test is maintained at approximately 35 pounds per acre. Medium initial soil phosphorus levels (Figure (1), Panel (b)), indicate that soil phosphorus levels are maintained with fairly constant phosphate applications. Low initial phosphorus levels (Figure (1), Panel (c)) are built up quickly with high phosphate application rates. Then, the rates are reduced sharply as the soil test approaches 40 pounds per acre.

Phosphate's relative price elasticity largely depends on the phosphorus soil test. Generally, with a Bray phosphorus test exceeding 40 pounds per acre, the elasticity is relatively high. That is, on soils with high phosphorus levels, price changes have large impacts on phosphate application rates. With a phosphorus test of 40, the relative price elasticity is about -2.0. Thus, a 10 percent increase in the phosphate price results in a 20 percent decrease in optimum P_2O_5 application rates. As the phosphorus test approaches 30, the relative price elasticity approaches -1.0. On soils with low phosphorus levels, price changes have small effect on optimum application rates. For example, with a phosphorus soil test of 20, the relative price elasticity is about -0.5.

The results for wheat are quite similar to those for corn (Figure 2). The analysis assumes $3.30 per bushel for wheat. At each soil phosphorus level, phosphate application rates on wheat are about 5 pounds per acre less than those on corn. Phosphate's relative price elasticity is approximately the same as well: -2.0 with soil test of 40, -1.0 with a soil test of 30, and -0.5 with a soil test of 20 pounds per acre.

Generalizations are much different for soybeans (Figure 3). The price of soybeans is assumed to be $6.20 per bushel. With the soil test above 30 pounds per acre, no phosphate is applied. It pays to "mine" the soil of phosphorus until the soil test is under 30. Then phosphate is applied to maintain the soil test at about 25. Phosphates relative price elasticity is high. Above a soil test of 25, small price changes have large impacts on phosphate application rates. With a soil test of 25, the relative price elasticity is about -2.0. With a soil test of 20, it drops to about -0.5.

Conclusions

Phosphate application rate recommendations now reflect the soil's inherent levels of phosphorus. They neglect phosphate prices and product prices. This analysis provides a method for incorporating these prices in phosphate application rate recommendations. Also, the analysis indicates that economic optimum application rates may change dramatically with phosphate and/or output price changes.

Generally, the optimum phosphate application rate for corn and wheat is higher than it is for soybeans. Conversely, phosphate's relative price elasticity is higher for soybeans than it is for corn and wheat. For the majority of Corn Belt soils growing corn and wheat, a 10 percent increase (decrease) in the relative price of phosphate should lower (raise) optimum application rates by 10 to 20 percent.

While soybeans require less phosphate and may even allow "mining" of soil phosphorus, farmers' best strategy would be to maintain soil phosphorus levels near those best suited for corn and wheat. Soybeans are usually rotated with corn and/or wheat, thus soybeans should not severely deplete phosphorus levels.

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Figure 2. Optimum P_2O_5 Application Rate for Wheat, Three Initial Soil Phosphorus Levels.

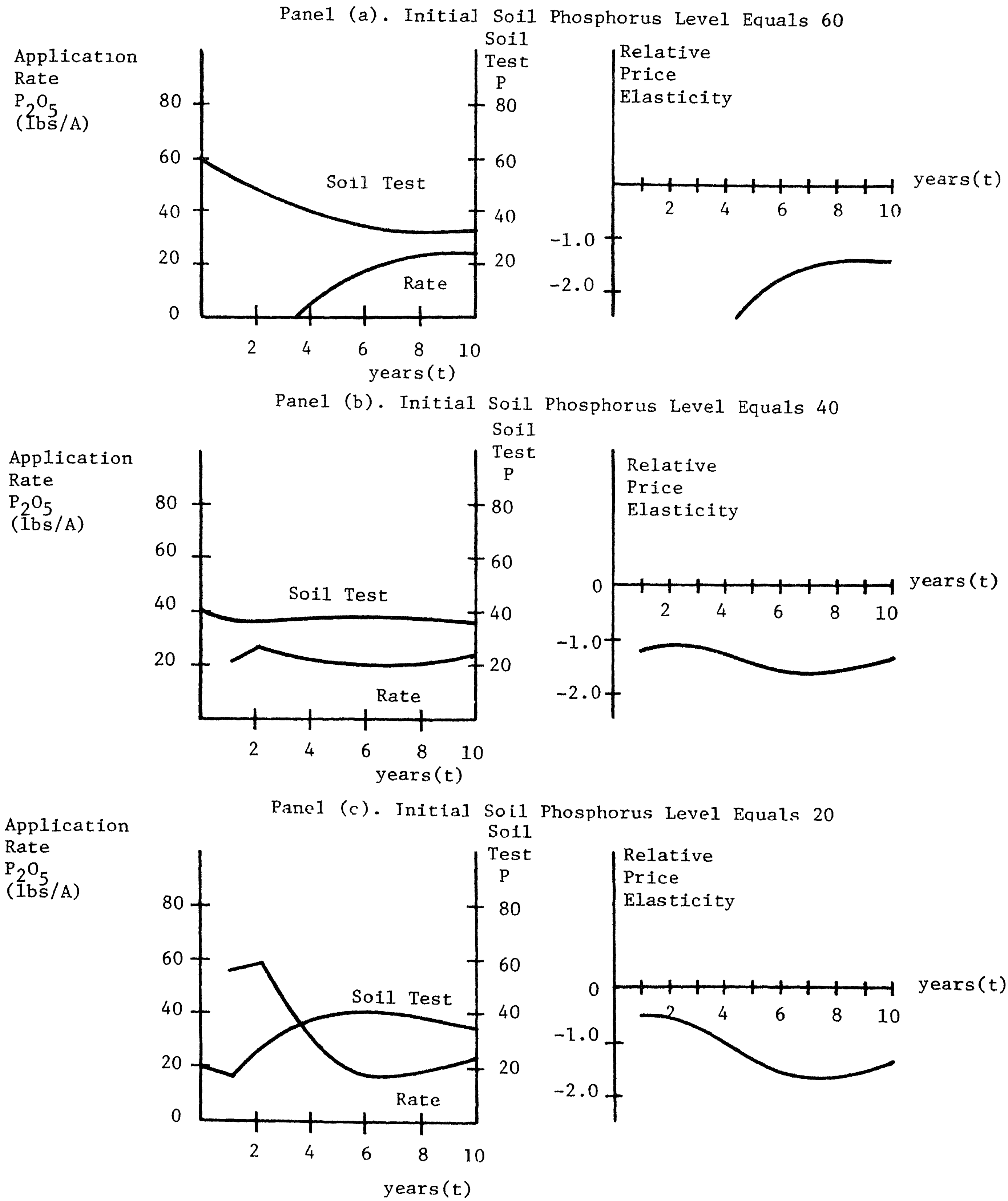


Figure 1. Optimum P_2O_5 Application Rate for Corn, Three Initial Soil Phosphorus Levels.

Panel (a). Initial Soil Phosphorus Level Equals 60

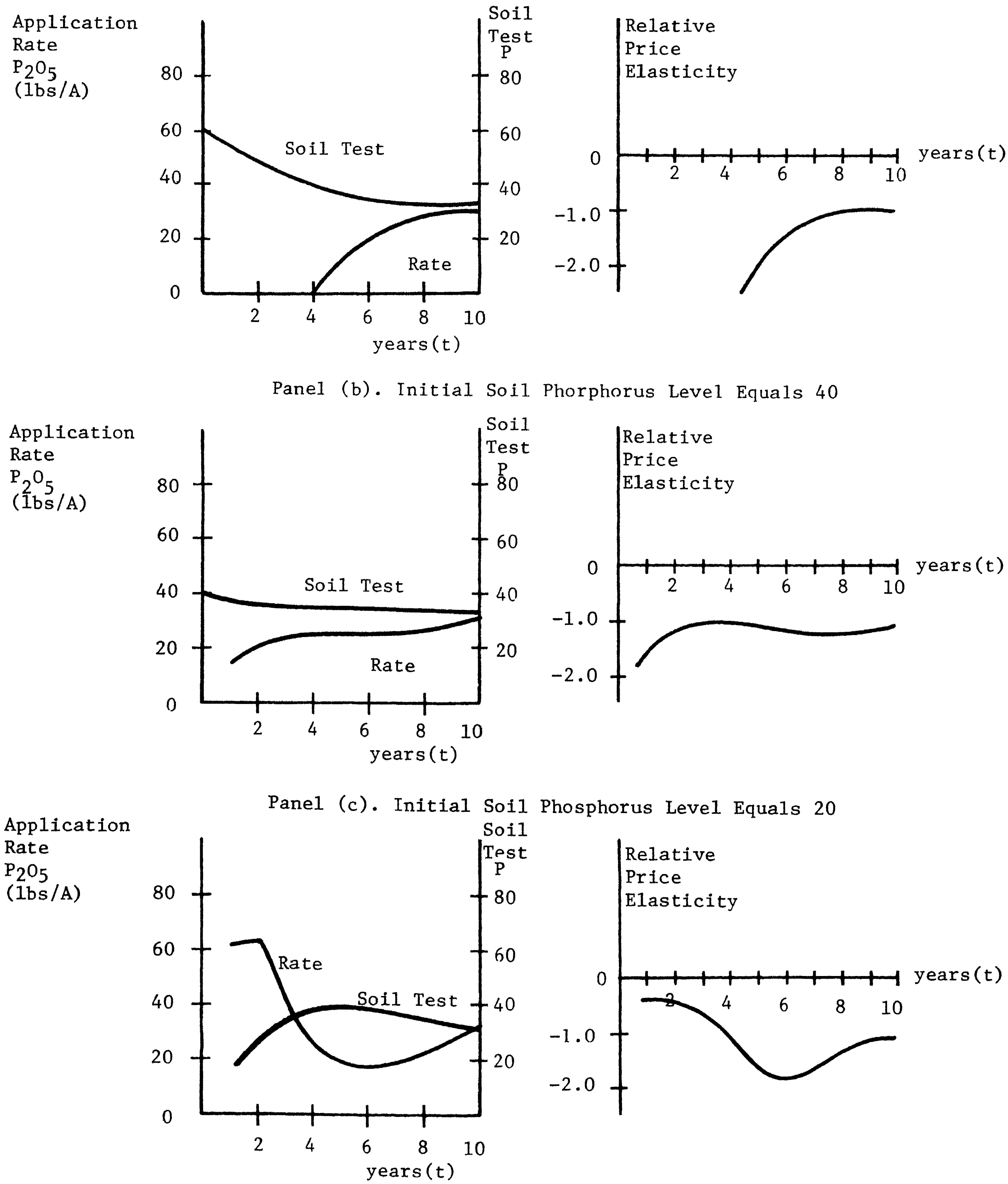
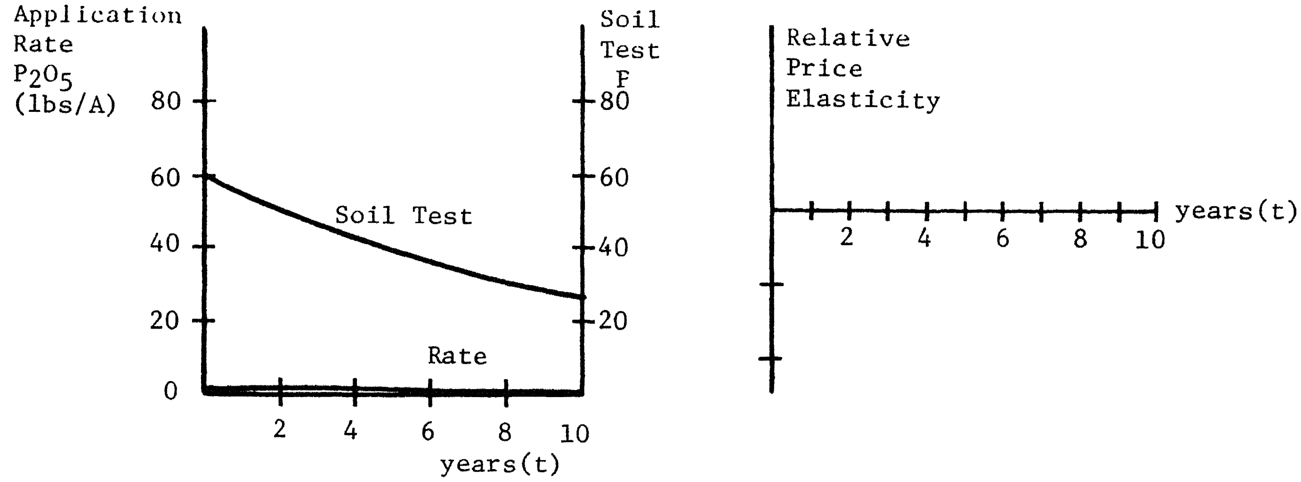
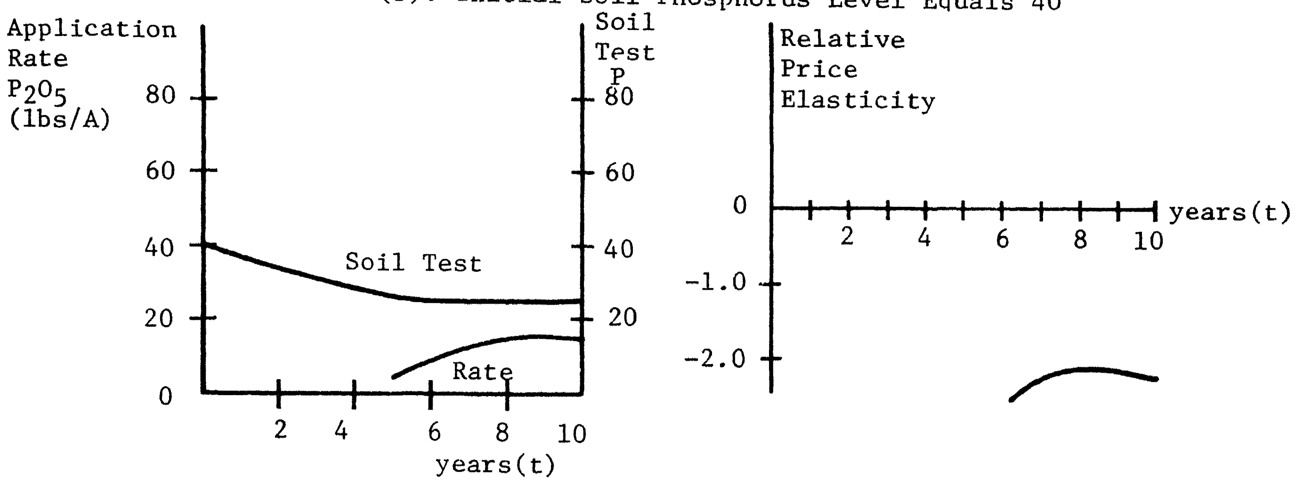


Figure 3. Optimum P_2O_5 Application Rate for Soybeans, Three Initial Soil Phosphorus Levels.

Panel (a). Initial Soil Phosphorus Level Equals 60



Panel (b). Initial Soil Phosphorus Level Equals 40



Panel (c). Initial Soil Phosphorus Level Equals 20